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Direct method for the periodic amplification of a soliton in an optical fibre link with loss

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Abstract
A direct approach is applied to the periodic amplification of a soliton in an optical fibre link with loss. In a single soliton case, the adiabatic solution and first-order correction are given for the system. The apparent advantage of this direct approach is that it not only presents the slow evolution of soliton parameters, but also the perturbation-induced radiation, and can be easily used to investigate the system of dispersion management with periodically varying dispersion and other fields.

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1. Introduction

The physical situations that give rise to the standard soliton equations tend to be highly idealized. In more practical systems, to the governing equations have to be added some small terms that are called perturbation. Thus perturbation theory is important to treat these nearly integrable systems [1]. Several soliton perturbation theories have been developed to study the influences of small perturbations on integrable equations. In these theories, one of the most systematic perturbation methods in dealing with these problems is based on the inverse scattering transform (IST) which has been well studied [1–4]. Another way to study perturbation is the direct method based upon the theory of linear partial differential equations, and many authors have applied it to various fields [5–8]. In general, the results of these two perturbation theories, based on IST and the direct method, respectively, are consistent. Recently, the direct approach based on a complete set of the squared Jost functions has been proposed and further developed [9–13]. As emphasized by them, the direct method, in general, needs no knowledge of IST, however, results of IST are helpful in finding the complete set of squared Jost functions.

For the periodic amplification of a soliton in an optical fibre link with loss, several methods, such as Lie transformation and the averaging method, have been well adopted by many authors [14–19]. But these papers did not present the explicit expression for the rapidly varying portion of perturbation-induced radiation of the system caused by the periodic amplification. In this paper, we investigate this problem by using the direct method [9, 12, 13], and give the adiabatic solution (slowly varying portion) and first-order correction (rapidly varying portion) for the single soliton case. This method can easily be used to treat the system of dispersion management with periodically varying dispersion [20] and other fields.

This paper is organized as follows. In section 2, the model is reduced as a perturbed nonlinear equation with a small perturbation term. In section 3, the perturbed nonlinear equations are linearized by expanding their solution into the sum of the adiabatic solution and correction terms and at the same time derivative expansion for the space variable is always needed to eliminate the potential secular terms. The key is to find eigenfunctions of a linear operator associated with the linearized equation, namely, the so-called squared Jost functions, and show that those eigenfunctions form an orthogonal complete set. In section 4, the first-order corrections are expanded in terms of the squared Jost functions and the secularity conditions are presented. Then the adiabatic variation of the parameters is determined by the secularity conditions in section 5. In section 6, the first-order correction is given. Conclusions are summarized in section 7.

2. Reduction of equation

The periodic amplification of a soliton in an optical fibre link with loss can be described by adding a gain term to the usual nonlinear Schrödinger (NLS) equation [14]

$$\mathrm{i}\frac{\partial u}{\partial Z}+\frac{1}{2}\frac{\partial^2 u}{\partial T^2}+|u|^2u=r_1[u] \tag{1}$$

where

$$r_1[u]=-\mathrm{i}\frac{1}{2}\Gamma u+\mathrm{i}(\sqrt{G}-1)\sum_{m=1}^{N_A}\delta(Z-mZ_A)u. \tag{2}$$

N_A is the total number of amplifiers, $Z_A=z_A/z_0$ is the period normalized to the dispersion distance z_0 (z_A is the practical period of the amplifiers in the link), $\Gamma=\gamma z_0$ is the normalized loss rate and γ $(=\omega_1\,\mathrm{Im}\{\chi_1\}/c)$ is the loss rate per unit length of the fibre and $G=\mathrm{e}^{\Gamma Z_A}$ is the amplifier gain needed to compensate for the fibre loss. The Dirac δ-function $\delta(Z-mZ_A)$ accounts for the lumped nature of the amplification at location $Z=mZ_A$. The factor $\sqrt{G}-1$ represents the change in soliton amplitude during amplification.

Because of rapid variation in soliton energy introduced by the lumped-amplification scheme, it is useful to make the transformation

$$u(Z,T)=\beta(Z)v(Z,T) \tag{3}$$

where $\beta(Z)$ indicates the part of rapid variation and $v(Z,T)$ is a slowly varying function of Z. By substituting (3) into equation (1), the slowly varying function $v(Z,T)$ is found to satisfy

$$\mathrm{i}\frac{\partial v}{\partial Z}+\frac{1}{2}\frac{\partial^2 v}{\partial T^2}+\beta^2(Z)|v|^2v=0 \tag{4}$$

where the rapid part $\beta(Z)$ is obtained by solving

$$\frac{\partial\beta}{\partial Z}=-\frac{1}{2}\Gamma\beta+(\sqrt{G}-1)\sum_{m=1}^{N_A}\delta(Z-mZ_A)\beta. \tag{5}$$

We can easily find that the $\beta(Z)$ is of the expression from equation (5)

$$\beta(Z) = \beta(0) \sum_{m=0}^{N_A-1} \mathrm{e}^{-\frac{1}{2}\Gamma(Z-mZ_A)}(H(Z-mZ_A) - H(Z-(m+1)Z_A)) + \beta(0)\,\mathrm{e}^{-\frac{1}{2}\Gamma(Z-N_AZ_A)}H(Z-N_AZ_A) \tag{6}$$

and is the periodic function with the period Z_A, where $H(Z)$ is the Heaviside function, that is, $H(Z) = 0$ as $Z \leqslant 0$; while $H(Z) = 1$ as $Z > 0$.

The concept of an average soliton makes use of the fact that $\beta^2(Z)$ in equation (4) varies rapidly with a small period $Z_A \ll 1$. Since an optical soliton changes only a little in a short distance Z_A, one can replace $\beta^2(Z)$ by its average value over one period $\langle\beta^2(Z)\rangle$,

$$\langle\beta^2(Z)\rangle = \frac{1}{Z_A}\int_0^{Z_A} \mathrm{d}Z\,\beta^2(0)\,\mathrm{e}^{-\Gamma Z} = \beta^2(0)\frac{G-1}{G\ln G}. \tag{7}$$

Assuming that the input peak power of the average soliton can be chosen such that $\langle\beta^2(Z)\rangle = 1$, thus equation (7) leads to

$$\beta^2(0) = \frac{G\ln G}{G-1} \tag{8}$$

and one finds that equation (4) becomes

$$\mathrm{i}\frac{\partial v}{\partial Z} + \frac{1}{2}\frac{\partial^2 v}{\partial T^2} + |v|^2 v = r[v] \tag{9}$$

where

$$r[v] = (1 - \beta^2(Z))|v|^2 v \tag{10}$$

and when Z_A is small,

$$1 - \beta^2(Z) = 1 - \frac{G\ln G}{G-1}\,\mathrm{e}^{-\Gamma Z} \qquad \text{as} \quad 0 < Z < Z_A \tag{11}$$

is also a small quantity. Thus we transform NLS equation (1) into a perturbed NLS equation with the small perturbation term $r[v]$ given by equation (10).

3. Linearization of the NLS equation

We now turn to the perturbed NLS equation (9) by the direct perturbation method. In treating equation (9) by the direct perturbation method, we first introduce a parameter ϵ to characterize the perturbation term and replace the term $r[v]$ by $\epsilon r[v]$. Thus equation (9) can be rewritten as

$$\mathrm{i}\frac{\partial v}{\partial Z} + \frac{1}{2}\frac{\partial^2 v}{\partial T^2} + |v|^2 v = \epsilon r[v] \tag{12}$$

where $r[v]$ is given by (10), which is a small quantity as Z_A is small. When $\epsilon = 0$, equation (12) is the standard NLS equation and under the vanishing boundary condition the bright soliton solutions can be found by a standard procedure of the IST [4]. The corresponding bright one-soliton solution is of the form

$$v_1(Z, T) = 2\nu\,\mathrm{sech}\,\theta\,\mathrm{e}^{-\mathrm{i}\varphi} \tag{13}$$

where

$$\theta = 2\nu(T + 2\mu Z - \tau_0) \tag{14}$$

$$\varphi = 2\mu T + 2(\mu^2 - \nu^2)Z + \varphi_0 \tag{15}$$

where $\nu > 0$, μ, τ_0 and φ_0 are the constants.

Now let us consider an approximation solution of equation (12) up to the first-order correction

$$v = w + \epsilon q \tag{16}$$

with the initial condition

$$v(Z = 0, T) = v_1(Z = 0, T). \tag{17}$$

According to the idea of adiabatic solutions, the zeroth-order approximation w is the so-called adiabatic solution which has the same form as the exact solution v_1 with the parameters μ, ν, τ_0 and φ_0 modulating on the $Z_1 = \epsilon Z$ scale [9, 13] and q represents the remaining term which is called the first-order correction. Introducing a two-scale expansion $\partial_Z = \partial_{Z_0} + \epsilon\partial_{Z_1}$ in which $Z_n = \epsilon^n Z$, $n = 0, 1$ are treated as two independent variables as usual, and substituting these into the perturbed NLS equation (12) and equating the coefficients of each power of ϵ, we obtain the following approximation equations:

$$\mathrm{i}\frac{\partial w}{\partial Z} + \frac{1}{2}\frac{\partial^2 w}{\partial T^2} + |w|^2 w = 0 \tag{18}$$

$$\mathrm{i}\frac{\partial q}{\partial Z} + \frac{1}{2}\frac{\partial^2 q}{\partial T^2} + 2|w|^2 q + w^2\overline{q} = R[w] \tag{19}$$

where $R[w] \equiv r[w] - \mathrm{i}\frac{\partial w}{\partial Z_1}$ is the effective source with the expression, in the case of a single soliton

$$R[w] = (1 - \beta^2(Z))8\nu^3 \operatorname{sech}^3\theta\, \mathrm{e}^{-\mathrm{i}\varphi} - (\mathrm{i}2\nu' - \mathrm{i}2\nu\theta' \tanh\theta + 2\nu\varphi') \operatorname{sech}\theta\, \mathrm{e}^{-\mathrm{i}\varphi} \tag{20}$$

and

$$\theta' = 2\nu'(T + 2\mu Z - \tau_0) + 2\nu(2\mu' Z - \tau_0') \tag{21}$$

$$\varphi' = 2\mu' T + 4(\mu\mu' - \nu\nu')Z + \varphi_0' \tag{22}$$

and we do not distinguish Z_0 and Z, and '$'$' denotes the derivative with respect to Z_1. Then the first-order approximation linearized NLS equation (19) and its complex conjugate, together with the initial condition $q(Z = 0, T) = 0$, can be reduced into the following form:

$$(\mathrm{i}\partial_Z - \mathbf{L}(w))\mathbf{Q} = \mathbf{R} \tag{23}$$

where the operator $\mathrm{i}\partial_Z - \mathbf{L}(w)$ is the so-called linearized operator

$$\mathbf{L}(w) = \begin{pmatrix} -\frac{1}{2}\partial_{TT} - 2|w|^2 & -w^2 \\ \overline{w}^2 & \frac{1}{2}\partial_{TT} + 2|w|^2 \end{pmatrix} \tag{24}$$

and

$$\mathbf{Q} = \begin{pmatrix} q \\ \overline{q} \end{pmatrix} \qquad \mathbf{R} = \begin{pmatrix} R[w] \\ -\overline{R[w]} \end{pmatrix}. \tag{25}$$

In a single soliton case, (24) reduces to

$$\mathbf{L}(w) = \begin{pmatrix} -\frac{1}{2}\partial_{TT} - 8\nu^2 \operatorname{sech}^2\theta & -4\nu^2 \operatorname{sech}^2\theta\, \mathrm{e}^{-\mathrm{i}2\varphi} \\ 4\nu^2 \operatorname{sech}^2\theta\, \mathrm{e}^{\mathrm{i}2\varphi} & \frac{1}{2}\partial_{TT} + 8\nu^2 \operatorname{sech}^2\theta \end{pmatrix} \tag{26}$$

and $\mathbf{R}$ is determined by (20).

For the linearized operator $\mathrm{i}\partial_Z - \mathbf{L}(w)$, we can verify that $\boldsymbol{\Psi}(Z,T,\lambda)$, $\boldsymbol{\Psi}(Z,T,\lambda_1)$, $\dot{\boldsymbol{\Psi}}(Z,T,\lambda_1)$, $\widetilde{\boldsymbol{\Psi}}(Z,T,\lambda)$, $\widetilde{\boldsymbol{\Psi}}(T,\overline{\lambda}_1)$ and $\dot{\widetilde{\boldsymbol{\Psi}}}(T,\overline{\lambda}_1)$ satisfy the eigen-equations in the following [9] (here λ is continuous spectral which is the real number and λ_1 is discrete spectral which is the complex number $\lambda_1 = \mu + \mathrm{i}\nu$):

$$(\mathrm{i}\partial_Z - \mathbf{L}(w))\boldsymbol{\Psi}(Z,T,\lambda) = 2\lambda^2\boldsymbol{\Psi}(Z,T,\lambda) \tag{27}$$

$$(\mathrm{i}\partial_Z - \mathbf{L}(w))\boldsymbol{\Psi}(Z,T,\lambda_1) = 2\lambda_1^2\boldsymbol{\Psi}(Z,T,\lambda_1) \tag{28}$$

$$(\mathrm{i}\partial_Z - \mathbf{L}(w))\dot{\boldsymbol{\Psi}}(Z,T,\lambda_1) = 4\lambda_1\boldsymbol{\Psi}(Z,T,\lambda_1) + 2\lambda_1^2\dot{\boldsymbol{\Psi}}(Z,T,\lambda_1) \tag{29}$$

$$(\mathrm{i}\partial_Z - \mathbf{L}(w))\widetilde{\boldsymbol{\Psi}}(Z,T,\lambda) = -2\lambda^2\widetilde{\boldsymbol{\Psi}}(Z,T,\lambda) \tag{30}$$

$$(\mathrm{i}\partial_Z - \mathbf{L}(w))\widetilde{\boldsymbol{\Psi}}(Z,T,\overline{\lambda}_1) = -2\overline{\lambda}_1^2\widetilde{\boldsymbol{\Psi}}(Z,T,\overline{\lambda}_1) \tag{31}$$

$$(\mathrm{i}\partial_Z - \mathbf{L}(w))\dot{\widetilde{\boldsymbol{\Psi}}}(Z,T,\overline{\lambda}_1) = -4\overline{\lambda}_1\widetilde{\boldsymbol{\Psi}}(Z,T,\overline{\lambda}_1) - 2\overline{\lambda}_1{}^2\dot{\widetilde{\boldsymbol{\Psi}}}(Z,T,\overline{\lambda}_1) \tag{32}$$

and form a complete set of the linearized operator (26), namely, the expression of unity

$$\begin{aligned}
\delta(T-T') = -\frac{1}{\pi}\int_{-\infty}^{+\infty}\mathrm{d}\lambda\,\Bigg\{&\frac{(\lambda-\overline{\lambda}_1)^2}{(\lambda-\lambda_1)^2}\boldsymbol{\Psi}(Z,T,\lambda)\boldsymbol{\Psi}^A(Z,T',\lambda)\\
&- \frac{(\lambda-\lambda_1)^2}{(\lambda-\overline{\lambda}_1)^2}\widetilde{\boldsymbol{\Psi}}(Z,T,\lambda)\widetilde{\boldsymbol{\Psi}}^A(Z,T',\lambda)\Bigg\} - 8\nu\boldsymbol{\Psi}(Z,T,\lambda_1)\boldsymbol{\Psi}^A(Z,T',\lambda_1)\\
&- \mathrm{i}8\nu^2\dot{\boldsymbol{\Psi}}(Z,T,\lambda_1)\boldsymbol{\Psi}^A(Z,T',\lambda_1) - \mathrm{i}8\nu^2\boldsymbol{\Psi}(Z,T,\lambda_1)\dot{\boldsymbol{\Psi}}^A(Z,T',\lambda_1)\\
&+ 8\nu\widetilde{\boldsymbol{\Psi}}(Z,T,\overline{\lambda}_1)\widetilde{\boldsymbol{\Psi}}^A(Z,T',\overline{\lambda}_1) - \mathrm{i}8\nu^2\widetilde{\boldsymbol{\Psi}}(Z,T,\overline{\lambda}_1)\dot{\widetilde{\boldsymbol{\Psi}}}^A(Z,T',\overline{\lambda}_1)\\
&- \mathrm{i}8\nu^2\dot{\widetilde{\boldsymbol{\Psi}}}(Z,T,\overline{\lambda}_1)\widetilde{\boldsymbol{\Psi}}^A(Z,T',\overline{\lambda}_1)
\end{aligned} \tag{33}$$

and have all the non-zero inner products, namely orthogonal relations

$$\langle\boldsymbol{\Psi}(\lambda')|\boldsymbol{\Psi}(\lambda)\rangle = -\pi\frac{(\lambda-\lambda_1)^2}{(\lambda-\overline{\lambda}_1)^2}\delta(\lambda-\lambda') \tag{34}$$

$$\langle\dot{\boldsymbol{\Psi}}(\lambda_1)|\boldsymbol{\Psi}(\lambda_1)\rangle = \langle\boldsymbol{\Psi}(\lambda_1)|\dot{\boldsymbol{\Psi}}(\lambda_1)\rangle = \mathrm{i}\frac{1}{8\nu^2} \tag{35}$$

$$\langle\dot{\boldsymbol{\Psi}}(\lambda_1)|\dot{\boldsymbol{\Psi}}(\lambda_1)\rangle = -\frac{1}{8\nu^3} \tag{36}$$

$$\langle\widetilde{\boldsymbol{\Psi}}(\lambda')|\widetilde{\boldsymbol{\Psi}}(\lambda)\rangle = \pi\frac{(\lambda-\overline{\lambda}_1)^2}{(\lambda-\lambda_1)^2}\delta(\lambda-\lambda') \tag{37}$$

$$\langle\dot{\widetilde{\boldsymbol{\Psi}}}(\overline{\lambda}_1)|\widetilde{\boldsymbol{\Psi}}(\overline{\lambda}_1)\rangle = \langle\widetilde{\boldsymbol{\Psi}}(\overline{\lambda}_1)|\dot{\widetilde{\boldsymbol{\Psi}}}(\overline{\lambda}_1)\rangle = \mathrm{i}\frac{1}{8\nu^2} \tag{38}$$

$$\langle\dot{\widetilde{\boldsymbol{\Psi}}}(\overline{\lambda}_1)|\dot{\widetilde{\boldsymbol{\Psi}}}(\overline{\lambda}_1)\rangle = \frac{1}{8\nu^3} \tag{39}$$

where $\boldsymbol{\Psi}(Z,T,\lambda)$, $\widetilde{\boldsymbol{\Psi}}(Z,T,\lambda)$ are called the squared Jost functions associated with the Jost functions [9], and $\boldsymbol{\Psi}(Z,T,\lambda)$, $\boldsymbol{\Psi}(Z,T,\lambda_1)$, $\dot{\boldsymbol{\Psi}}(Z,T,\lambda_1)$, $\widetilde{\boldsymbol{\Psi}}(Z,T,\lambda)$, $\widetilde{\boldsymbol{\Psi}}(Z,T,\overline{\lambda}_1)$ and $\dot{\widetilde{\boldsymbol{\Psi}}}(Z,T,\overline{\lambda}_1)$ are of the following expressions [4], respectively:

$$\boldsymbol{\Psi}(Z,T,\lambda) = \begin{pmatrix} -\frac{\nu^2}{(\lambda-\mu+\mathrm{i}\nu)^2}\operatorname{sech}^2\theta\,\mathrm{e}^{-2\mathrm{i}\varphi}\,\mathrm{e}^{2\mathrm{i}\lambda T}\\ -\frac{1}{(\lambda-\mu+\mathrm{i}\nu)^2}(\lambda-\mu+\mathrm{i}\nu\tanh\theta)^2\,\mathrm{e}^{2\mathrm{i}\lambda T}\end{pmatrix} \tag{40}$$

$$\mathbf{\Psi}(Z,T,\lambda_1)=\begin{pmatrix}\frac{1}{4}\operatorname{sech}^2\theta\,\mathrm{e}^{-2\mathrm{i}\varphi}\,\mathrm{e}^{2\mathrm{i}\lambda_1 T}\\ -\frac{1}{4}(1+\tanh\theta)^2\,\mathrm{e}^{2\mathrm{i}\lambda_1 T}\end{pmatrix} \tag{41}$$

$$\dot{\mathbf{\Psi}}(Z,T,\lambda_1)=\begin{pmatrix}\mathrm{i}\frac{1}{4}\left(\frac{1}{\nu}+2T\right)\operatorname{sech}^2\theta\,\mathrm{e}^{-2\mathrm{i}\varphi}\,\mathrm{e}^{2\mathrm{i}\lambda_1 T}\\ -\mathrm{i}\frac{1}{4}\left(\frac{1}{\nu}+2T\right)(1+\tanh\theta)^2\,\mathrm{e}^{2\mathrm{i}\lambda_1 T}+\mathrm{i}\frac{1}{2\nu}(1+\tanh\theta)\,\mathrm{e}^{2\mathrm{i}\lambda_1 T}\end{pmatrix} \tag{42}$$

$$\widetilde{\mathbf{\Psi}}(Z,T,\lambda)=\begin{pmatrix}\frac{1}{(\lambda-\mu-\mathrm{i}\nu)^2}(\lambda-\mu-\mathrm{i}\nu\tanh\theta)^2\,\mathrm{e}^{-2\mathrm{i}\lambda T}\\ \frac{\nu^2}{(\lambda-\mu-\mathrm{i}\nu)^2}\operatorname{sech}^2\theta\,\mathrm{e}^{2\mathrm{i}\varphi}\,\mathrm{e}^{-2\mathrm{i}\lambda T}\end{pmatrix} \tag{43}$$

$$\widetilde{\mathbf{\Psi}}(Z,T,\overline{\lambda}_1)=\begin{pmatrix}\frac{1}{4}(1+\tanh\theta)^2\,\mathrm{e}^{-2\mathrm{i}\overline{\lambda}_1 T}\\ -\frac{1}{4}\operatorname{sech}^2\theta\,\mathrm{e}^{2\mathrm{i}\varphi}\,\mathrm{e}^{-2\mathrm{i}\overline{\lambda}_1 T}\end{pmatrix} \tag{44}$$

$$\dot{\widetilde{\mathbf{\Psi}}}(Z,T,\overline{\lambda}_1)=\begin{pmatrix}-\mathrm{i}\frac{1}{4}\left(\frac{1}{\nu}+2T\right)(1+\tanh\theta)^2\,\mathrm{e}^{-2\mathrm{i}\overline{\lambda}_1 T}+\frac{\mathrm{i}}{2\nu}(1+\tanh\theta)\,\mathrm{e}^{-2\mathrm{i}\overline{\lambda}_1 T}\\ \mathrm{i}\frac{1}{4}\left(\frac{1}{\nu}+2T\right)\operatorname{sech}^2\theta\,\mathrm{e}^{2\mathrm{i}\varphi}\,\mathrm{e}^{-2\mathrm{i}\overline{\lambda}_1 T}\end{pmatrix} \tag{45}$$

and here we have used notation

$$\dot{\mathbf{\Psi}}(Z,T,\lambda_1)=\frac{\mathrm{d}\mathbf{\Psi}(Z,T,\lambda)}{\mathrm{d}\lambda}\Big|_{\lambda=\lambda_1} \tag{46}$$

and the corresponding inner product

$$\langle\mathbf{\Psi}(\lambda')|\mathbf{\Psi}(\lambda)\rangle=\int_{-\infty}^{+\infty}\mathrm{d}T\,\mathbf{\Psi}^A(Z,T,\lambda')\mathbf{\Psi}(Z,T,\lambda) \tag{47}$$

where the adjoint function related to $\mathbf{\Psi}^A(Z,T,\lambda)$ and $\widetilde{\mathbf{\Psi}}^A(Z,T,\lambda)$ are given by

$$\mathbf{\Psi}^A(Z,T,\lambda)=\left(-\frac{\nu^2}{(\lambda-\mu+\mathrm{i}\nu)^2}\operatorname{sech}^2\theta\,\mathrm{e}^{2\mathrm{i}\varphi}\,\mathrm{e}^{-2\mathrm{i}\lambda T},\quad \frac{1}{(\lambda-\mu+\mathrm{i}\nu)^2}(\lambda-\mu-\mathrm{i}\nu\tanh\theta)^2\,\mathrm{e}^{-2\mathrm{i}\lambda T}\right) \tag{48}$$

$$\widetilde{\mathbf{\Psi}}^A(Z,T,\lambda)=\left(\frac{1}{(\lambda-\mu-\mathrm{i}\nu)^2}(\lambda-\mu+\mathrm{i}\nu\tanh\theta)^2\,\mathrm{e}^{\mathrm{i}2\lambda T},\quad -\frac{\nu^2}{(\lambda-\mu-\mathrm{i}\nu)^2}\operatorname{sech}^2\theta\,\mathrm{e}^{-\mathrm{i}2\varphi}\,\mathrm{e}^{\mathrm{i}2\lambda T}\right). \tag{49}$$

4. Expansion in terms of the squared Jost functions

From the expression of unity (33), we can express the first-order approximation solution $\mathbf{Q}(Z,T)$ as

$$\begin{aligned}\mathbf{Q}(Z,T)=\frac{1}{\pi}\int_{-\infty}^{+\infty}&\mathrm{d}\lambda\{f(Z,\lambda)\mathbf{\Psi}(Z,T,\lambda)+\widetilde{f}(Z,\lambda)\widetilde{\mathbf{\Psi}}(Z,T,\lambda)\}+f_1(Z)\mathbf{\Psi}(Z,T,\lambda_1)\\ &+g_1(Z)\dot{\mathbf{\Psi}}(Z,T,\lambda_1)+\widetilde{f}_1(Z)\widetilde{\mathbf{\Psi}}(Z,T,\overline{\lambda}_1)+\widetilde{g}_1(Z)\dot{\widetilde{\mathbf{\Psi}}}(Z,T,\overline{\lambda}_1)\end{aligned} \tag{50}$$

provided that $\mathbf{Q}(Z,T)$ is continuous with respect to T, and $f(Z,\lambda)$, $\widetilde{f}(Z,\lambda)$, $f_1(Z)$, $\widetilde{f}_1(Z)$, $g_1(Z)$ and $\widetilde{g}_1(Z)$ with the relations

$$\widetilde{f}(Z,\lambda)=-\overline{f(Z,\lambda)}\qquad \widetilde{f}_1(Z)=-\overline{f_1(Z)}\qquad \widetilde{g}_1(Z)=-\overline{g_1(Z)} \tag{51}$$

which are determined later. Acting the operator $(\mathrm{i}\partial_Z-\mathbf{L}(w))$ on both sides of equation (50), one can find the following form with the aid of equations (27)–(32) and using equation (23)

$$\begin{aligned}\mathbf{R}=\frac{1}{\pi}\int_{-\infty}^{+\infty}&\mathrm{d}\lambda\{(\mathrm{i}f_Z+2\lambda^2 f)\mathbf{\Psi}(Z,T,\lambda)+(\mathrm{i}\widetilde{f}_Z-2\lambda^2\widetilde{f})\widetilde{\mathbf{\Psi}}(Z,T,\lambda)\}\\ &+\left(\mathrm{i}f_{1,Z}+2\lambda_1^2 f_1+4\lambda_1 g_1\right)\mathbf{\Psi}(Z,T,\lambda_1)+\left(\mathrm{i}g_{1,Z}+2\lambda_1^2 g_1\right)\dot{\mathbf{\Psi}}(Z,T,\lambda_1)\\ &+\left(\mathrm{i}\widetilde{f}_{1,Z}-2\overline{\lambda}_1^2\widetilde{f}_1-4\overline{\lambda}_1\widetilde{g}_1\right)\widetilde{\mathbf{\Psi}}(Z,T,\overline{\lambda}_1)+\left(\mathrm{i}\widetilde{g}_{1,Z}-2\overline{\lambda}_1^2\widetilde{g}_1\right)\dot{\widetilde{\mathbf{\Psi}}}(Z,T,\overline{\lambda}_1).\end{aligned} \tag{52}$$

Multiplying equation (52) by $\boldsymbol{\Psi}^A(Z, T, \lambda_1)$, $\dot{\boldsymbol{\Psi}}^A(Z, T, \lambda_1)$ and $\boldsymbol{\Psi}^A(Z, T, \lambda)$ successively, and integrating over T, we obtain the following ordinary differential equations with the aid of the orthogonal relations (34)–(39):

$$\left(\mathrm{i}g_{1,Z} + 2\lambda_1^2 g_1\right) \mathrm{i}\frac{1}{8\nu^2} = \langle \boldsymbol{\Psi}(\lambda_1)|\mathbf{R}\rangle \tag{53}$$

$$\left(\mathrm{i}f_{1,Z} + 2\lambda_1^2 f_1 + 4\lambda_1 g_1\right) \mathrm{i}\frac{1}{8\nu^2} - \left(\mathrm{i}g_{1,Z} + 2\lambda_1^2 g_1\right) \frac{1}{8\nu^3} = \langle \dot{\boldsymbol{\Psi}}(\lambda_1)|\mathbf{R}\rangle \tag{54}$$

$$-(\mathrm{i}f_Z + 2\lambda^2 f)\frac{(\lambda-\lambda_1)^2}{(\lambda-\overline{\lambda}_1)^2} = \langle \boldsymbol{\Psi}(\lambda)|\mathbf{R}\rangle. \tag{55}$$

Similarly, multiplying equation (52) by $\widetilde{\boldsymbol{\Psi}}^A(Z, T, \overline{\lambda}_1)$, $\dot{\widetilde{\boldsymbol{\Psi}}}^A(Z, T, \overline{\lambda}_1)$ and $\widetilde{\boldsymbol{\Psi}}^A(Z, T, \lambda)$ successively, and integrating over T, we obtain the following ordinary differential equations

$$\left(\mathrm{i}\widetilde{g}_{1,Z} - 2\overline{\lambda}_1^2 \widetilde{g}_1\right) \mathrm{i}\frac{1}{8\nu^2} = \langle \widetilde{\boldsymbol{\Psi}}(\overline{\lambda}_1)|\mathbf{R}\rangle \tag{56}$$

$$\left(\mathrm{i}\widetilde{f}_{1,Z} - 2\overline{\lambda}_1^2 \widetilde{f}_1 - 4\overline{\lambda}_1 \widetilde{g}_1\right) \mathrm{i}\frac{1}{8\nu^2} + \left(\mathrm{i}\widetilde{g}_{1,Z} - 2\overline{\lambda}_1^2 \widetilde{g}_1\right) \frac{1}{8\nu^3} = \langle \dot{\widetilde{\boldsymbol{\Psi}}}(\overline{\lambda}_1)|\mathbf{R}\rangle \tag{57}$$

$$(\mathrm{i}\widetilde{f}_Z - 2\lambda^2 \widetilde{f})\frac{(\lambda-\overline{\lambda}_1)^2}{(\lambda-\lambda_1)^2} = \langle \widetilde{\boldsymbol{\Psi}}(\lambda)|\mathbf{R}\rangle. \tag{58}$$

For ordinary differential equations (53), (54), (56) and (57), although the initial values of g_1, f_1, $\widetilde{g}_1$ and $\widetilde{f}_1$ vanish, their values may go to infinity if the right-hand sides of equations (53), (54), (56) and (57) are non-vanishing. So we must demand the right-hand sides of these equations be vanishing, namely, requiring secularity conditions

$$\langle \boldsymbol{\Psi}(\lambda_1)|\mathbf{R}\rangle = 0 \tag{59}$$

$$\langle \dot{\boldsymbol{\Psi}}(\lambda_1)|\mathbf{R}\rangle = 0 \tag{60}$$

$$\langle \widetilde{\boldsymbol{\Psi}}(\overline{\lambda}_1)|\mathbf{R}\rangle = 0 \tag{61}$$

$$\langle \dot{\widetilde{\boldsymbol{\Psi}}}(\overline{\lambda}_1)|\mathbf{R}\rangle = 0 \tag{62}$$

which ensure g_1, f_1, $\widetilde{g}_1$ and $\widetilde{f}_1$ are finite, where (59) and (60) are complex conjugates of (61) and (62). By means of them, the adiabatic solution can be determined. After the adiabatic solution is determined, the right-hand sides of (55) and (58) are known, thus f and $\widetilde{f}$ can be determined.

5. The adiabatic variation of the parameters

In this section, we will discuss the adiabatic variation of the parameters μ, ν, τ and κ by making use of the secularity conditions (59) and (60). First, by (20)–(22) with the aid of (14) and (15), we have

$$R[w] = (1-\beta^2(Z))8\nu^3 \operatorname{sech}^3\theta\, \mathrm{e}^{-\mathrm{i}\varphi} - (-\mathrm{i}2\nu'\theta\tanh\theta + A\tanh\theta + 2\mu'\theta + B)\operatorname{sech}\theta\, \mathrm{e}^{-\mathrm{i}\varphi} \tag{63}$$

where

$$A = -\mathrm{i}4\nu^2(2\mu' Z - \tau_0') \tag{64}$$

$$B = -8\nu^2\nu' Z + 4\nu\mu'\tau + 2\nu\varphi_0' + \mathrm{i}2\nu' \tag{65}$$

are independent of T. Thus from the secularity condition (59), we find by directly calculating

$$\langle \boldsymbol{\Psi}(\lambda_1)|\mathbf{R}\rangle = \int_{-\infty}^{+\infty} \boldsymbol{\Psi}^A(Z, T, \lambda_1)\mathbf{R}\,\mathrm{d}T = -\frac{1}{2\nu}\,\mathrm{e}^C(\mu' + \mathrm{i}\nu') = 0 \tag{66}$$

here we have used

$$-2\mathrm{i}\lambda_1 T + \mathrm{i}\varphi = \theta + C \tag{67}$$

and

$$C = 2\mathrm{i}(\mu + \mathrm{i}\nu)^2 Z + 2\nu\tau_0 + \mathrm{i}\varphi_0 \tag{68}$$

which is independent of T. Hence the secularity condition (59) yields

$$\mu = \mu_0 \qquad \nu = \nu_0 \tag{69}$$

where μ_0 and ν_0 are constants. This means there is no adiabatic variation of the parameters ν and μ.

We now turn to the second secularity condition (60). By calculating directly and noting (67)–(69), we have

$$\langle \dot{\boldsymbol{\Psi}}(\lambda_1)|\mathbf{R}\rangle = \int_{-\infty}^{+\infty} \dot{\boldsymbol{\Psi}}^A(Z, T, \lambda_1)\mathbf{R}\,\mathrm{d}T = \frac{1}{2\nu}\,\mathrm{e}^C\big(\mathrm{i}4\nu_0^2(1 - \beta^2(Z)\big) - \mathrm{i}\varphi_0' - 2\nu_0\tau_0'\big) = 0 \tag{70}$$

where we used

$$\frac{1}{\nu} - 2T = -\frac{1}{\nu}\theta + D \tag{71}$$

and

$$D = \frac{1}{\nu} + 2(2\mu Z - \tau_0) \tag{72}$$

which is independent of T. Hence the secularity condition (60) yields

$$\tau_0 = \tau_{01} \tag{73}$$

$$\begin{aligned}\varphi_0(Z) = 4\nu_0^2 \sum_{m=0}^{N_A-1} &\left((Z - mZ_A) - \beta^2(0)\frac{1}{\Gamma}(1 - \mathrm{e}^{-\Gamma(Z-mZ_A)})\right)(H(Z - mZ_A) \\ &- H(Z - (m+1)Z_A)) + 4\nu_0^2\bigg((Z - N_A Z_A) \\ &- \beta^2(0)\frac{1}{\Gamma}(1 - \mathrm{e}^{-\Gamma(Z-N_A Z_A)})\bigg) H(Z - N_A Z_A) + H(Z)\varphi_{01}\end{aligned} \tag{74}$$

where τ_{01} and φ_{01} are constants. Thus we can obtain the expression for the adiabatic solution from (13)

$$w(Z, T) = 2\nu_0\,\mathrm{sech}(2\nu_0(T + 2\mu_0 Z - \tau_{01}))\,\mathrm{e}^{-\mathrm{i}(2\mu_0 T + 2(\mu_0^2 - \nu_0^2)Z + \varphi_0)} \tag{75}$$

where φ_0 is given by (74). This means that the periodic amplification can cause the periodic variation of phase in the adiabatic solution.

6. Correction in the continuous spectrum

In section 5, in order to eliminate the secularities of the first-order approximation, it is required that the adiabatic variation of parameters satisfies (69), (73) and (74). Under these conditions, from the ordinary differential equations (53), (54), (56) and (57) with zero-initial condition, we can obtain

$$g_1(Z) = \widetilde{g}_1(Z) = f_1(Z) = \widetilde{f}_1(Z) = 0. \tag{76}$$

Thus, expression (50) becomes

$$\mathbf{Q}(Z,T)=\frac{1}{\pi}\int_{-\infty}^{+\infty}\mathrm{d}\lambda\{f(Z,\lambda)\mathbf{\Psi}(Z,T,\lambda)+\widetilde{f}(Z,\lambda)\widetilde{\mathbf{\Psi}}(Z,T,\lambda)\} \tag{77}$$

where $f(Z,\lambda)$ and $\widetilde{f}(Z,\lambda)$ with relations (51) satisfy equations (55) and (58), respectively, namely,

$$-(\mathrm{i}f_Z+2\lambda^2 f)=\frac{(\lambda-\overline{\lambda}_1)^2}{(\lambda-\lambda_1)^2}\langle\mathbf{\Psi}(\lambda)|\mathbf{R}\rangle \tag{78}$$

and

$$(\mathrm{i}\widetilde{f}_Z-2\lambda^2\widetilde{f})=\frac{(\lambda-\lambda_1)^2}{(\lambda-\overline{\lambda}_1)^2}\langle\widetilde{\mathbf{\Psi}}(\lambda)|\mathbf{R}\rangle. \tag{79}$$

In the following, we will solve equations (78) and (79). First, from (63)–(65) and using (69), (73) and (74), we have

$$R[w]=8\nu_0^3(1-\beta^2(Z))(\operatorname{sech}^3\theta-\operatorname{sech}\theta)\,\mathrm{e}^{-\mathrm{i}\varphi}. \tag{80}$$

Thus we obtain

$$\langle\mathbf{\Psi}(\lambda)|\mathbf{R}\rangle=\int_{-\infty}^{+\infty}\mathbf{\Psi}^A(Z,T,\lambda)\mathbf{R}\,\mathrm{d}T=-\frac{\pi}{F}(\lambda-\lambda_1)^2(1-\beta^2(Z))\,\mathrm{e}^E \tag{81}$$

where

$$E=-\mathrm{i}2\left(-2\lambda\mu_0+\mu_0^2+\nu_0^2\right)Z+\mathrm{i}\varphi_0(Z)-\mathrm{i}2(\lambda-\mu_0)\tau_{01} \tag{82}$$

is dependent of Z, and

$$F=\cosh\pi\frac{(\mu_0-\lambda)}{2\nu_0}. \tag{83}$$

Substituting (81) into (78), equation (78) becomes

$$f_Z-\mathrm{i}2\lambda^2 f=-\mathrm{i}\frac{\pi}{F}(\lambda-\overline{\lambda}_1)^2(1-\beta^2(Z))\,\mathrm{e}^E. \tag{84}$$

Solving equation (84) by Laplace transformation and taking $\varphi_0(Z)\approx\varphi_{01}$ as Z_A is small, we obtain the expression for $f(Z,\lambda)$ as follows:

$$\begin{aligned}f(Z,\lambda)=\Lambda&\sum_{m=0}^{N_A-1}\Omega(Z_A)\,\mathrm{e}^{\mathrm{i}2\lambda^2Z-\mathrm{i}2(\lambda-\lambda_1)(\lambda-\overline{\lambda}_1)mZ_A}H(Z-(m+1)Z_A)\\&+\Lambda\sum_{m=0}^{N_A-1}\Omega(Z-mZ_A)\,\mathrm{e}^{\mathrm{i}2\lambda^2Z-\mathrm{i}2(\lambda-\lambda_1)(\lambda-\overline{\lambda}_1)mZ_A}(H(Z-mZ_A)\\&-H(Z-(m+1)Z_A))+\Lambda\Omega(Z-N_AZ_A)\\&\times\mathrm{e}^{\mathrm{i}2\lambda^2Z-\mathrm{i}2(\lambda-\lambda_1)(\lambda-\overline{\lambda}_1)N_AZ_A}H(Z-N_AZ_A)\end{aligned} \tag{85}$$

where

$$\Omega(Z)=\frac{1-\mathrm{e}^{-\mathrm{i}2(\lambda-\lambda_1)(\lambda-\overline{\lambda}_1)Z}}{\mathrm{i}2(\lambda-\lambda_1)(\lambda-\overline{\lambda}_1)}-\beta^2(0)\frac{1-\mathrm{e}^{-(\mathrm{i}2(\lambda-\lambda_1)(\lambda-\overline{\lambda}_1)+\Gamma)Z}}{\mathrm{i}2(\lambda-\lambda_1)(\lambda-\overline{\lambda}_1)+\Gamma} \tag{86}$$

$$\Lambda=-\mathrm{i}\frac{\pi}{F}\,\mathrm{e}^{-\mathrm{i}2(\lambda-\mu_0)\tau_{01}+\mathrm{i}\varphi_{01}}(\lambda-\overline{\lambda}_1)^2 \tag{87}$$

and used $f(Z=0,\lambda)=0$.

Finally, from (77) , we can present the first-order approximation solution

$$q(Z,T) = -\frac{1}{\pi}\nu_0^2\,\mathrm{sech}^2\theta\,\mathrm{e}^{-2\mathrm{i}\varphi} I_1 - \frac{1}{\pi} I_2 + \frac{1}{\pi} 2\mathrm{i}\nu_0 \tanh\theta\, I_3 + \frac{1}{\pi}\nu_0^2 \tanh^2\theta\, I_4 \tag{88}$$

where

$$I_1 = \int_{-\infty}^{+\infty} f(Z,\lambda)\frac{1}{(\lambda-\overline{\lambda}_1)^2}\,\mathrm{e}^{2\mathrm{i}\lambda T}\,\mathrm{d}\lambda \tag{89}$$

$$I_2 = \int_{-\infty}^{+\infty} \overline{f(Z,\lambda)}\frac{(\lambda-\mu_0)^2}{(\lambda-\lambda_1)^2}\,\mathrm{e}^{-2\mathrm{i}\lambda T}\,\mathrm{d}\lambda \tag{90}$$

$$I_3 = \int_{-\infty}^{+\infty} \overline{f(Z,\lambda)}\frac{(\lambda-\mu_0)}{(\lambda-\lambda_1)^2}\,\mathrm{e}^{-2\mathrm{i}\lambda T}\,\mathrm{d}\lambda \tag{91}$$

$$I_4 = \int_{-\infty}^{+\infty} \overline{f(Z,\lambda)}\frac{1}{(\lambda-\lambda_1)^2}\,\mathrm{e}^{-2\mathrm{i}\lambda T}\,\mathrm{d}\lambda \tag{92}$$

where $f(Z,\lambda)$ is given by (85). Substituting (85) into (89)–(92), we have

$$\begin{aligned} I_1 = {} & -\mathrm{i}\pi \sum_{m=0}^{N_A-1} \int_{-\infty}^{+\infty} \frac{1}{F} G(Z_A,T,\lambda,m)\,\mathrm{e}^{\mathrm{i}2\lambda^2 Z}\,\mathrm{d}\lambda \times H(Z-(m+1)Z_A) \\ & -\mathrm{i}\pi \sum_{m=0}^{N_A-1} \int_{-\infty}^{+\infty} \frac{1}{F} G(Z-mZ_A,T,\lambda,m)\,\mathrm{e}^{\mathrm{i}2\lambda^2 Z}\,\mathrm{d}\lambda \\ & \times (H(Z-mZ_A) - H(Z-(m+1)Z_A)) \\ & -\mathrm{i}\pi \int_{-\infty}^{+\infty} \frac{1}{F} G(Z-N_A Z_A,T,\lambda,N_A)\,\mathrm{e}^{\mathrm{i}2\lambda^2 Z}\,\mathrm{d}\lambda \times H(Z-N_A Z_A) \end{aligned} \tag{93}$$

$$\begin{aligned} I_2 = {} & \mathrm{i}\pi \sum_{m=0}^{N_A-1} \int_{-\infty}^{+\infty} \frac{(\lambda-\mu_0)^2}{F} \overline{G(Z_A,T,\lambda,m)}\,\mathrm{e}^{-\mathrm{i}2\lambda^2 Z}\,\mathrm{d}\lambda \times H(Z-(m+1)Z_A) \\ & +\mathrm{i}\pi \sum_{m=0}^{N_A-1} \int_{-\infty}^{+\infty} \frac{(\lambda-\mu_0)^2}{F} \overline{G(Z-mZ_A,T,\lambda,m)}\,\mathrm{e}^{-\mathrm{i}2\lambda^2 Z}\,\mathrm{d}\lambda \\ & \times (H(Z-mZ_A) - H(Z-(m+1)Z_A)) \\ & +\mathrm{i}\pi \int_{-\infty}^{+\infty} \frac{(\lambda-\mu_0)^2}{F} \overline{G(Z-N_A Z_A,T,\lambda,N_A)}\,\mathrm{e}^{-\mathrm{i}2\lambda^2 Z}\,\mathrm{d}\lambda \times H(Z-N_A Z_A) \end{aligned} \tag{94}$$

$$\begin{aligned} I_3 = {} & \mathrm{i}\pi \sum_{m=0}^{N_A-1} \int_{-\infty}^{+\infty} \frac{(\lambda-\mu_0)}{F} \overline{G(Z_A,T,\lambda,m)}\,\mathrm{e}^{-\mathrm{i}2\lambda^2 Z}\,\mathrm{d}\lambda \times H(Z-(m+1)Z_A) \\ & +\mathrm{i}\pi \sum_{m=0}^{N_A-1} \int_{-\infty}^{+\infty} \frac{(\lambda-\mu_0)}{F} \overline{G(Z-mZ_A,T,\lambda,m)}\,\mathrm{e}^{-\mathrm{i}2\lambda^2 Z}\,\mathrm{d}\lambda \\ & \times (H(Z-mZ_A) - H(Z-(m+1)Z_A)) \\ & +\mathrm{i}\pi \int_{-\infty}^{+\infty} \frac{(\lambda-\mu_0)}{F} \overline{G(Z-N_A Z_A,T,\lambda,N_A)}\,\mathrm{e}^{-\mathrm{i}2\lambda^2 Z}\,\mathrm{d}\lambda \times H(Z-N_A Z_A) \end{aligned} \tag{95}$$

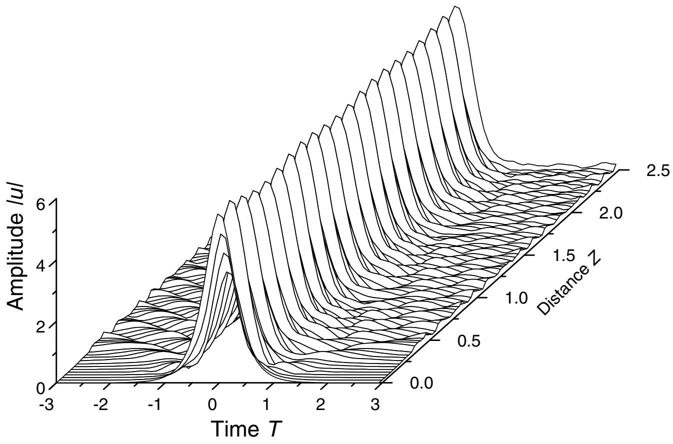

Figure 1. The behaviour of the amplitude $|u(Z,T)|$ of the approximation solution with the parameters: loss rate $\gamma = 0.028\,\text{km}^{-1}$, dispersion distance $z_0 = 404.93$ km and $\Gamma = 11.183$, $Z_A = 0.125$. $\nu_0 = 2$, $\mu_0 = \tau_{01} = \varphi_{01} = 0$.

and

$$I_4 = \overline{I_1} \tag{96}$$

where

$$G(Z,T,\lambda,m) = \frac{\exp \mathrm{i}\Theta - \exp \mathrm{i}\Phi}{\mathrm{i}2(\lambda-\lambda_1)(\lambda-\overline{\lambda}_1)} - \beta^2(0)\frac{\exp \mathrm{i}\Theta - \mathrm{e}^{-\Gamma Z}\exp \mathrm{i}\Phi}{\mathrm{i}2(\lambda-\lambda_1)(\lambda-\overline{\lambda}_1)+\Gamma} \tag{97}$$

and

$$\Theta(T,\lambda,m) = 2\lambda T - 2(\lambda-\lambda_1)(\lambda-\overline{\lambda}_1)mZ_A - 2(\lambda-\mu_0)\tau_{01} + \varphi_{01} \tag{98}$$

$$\Phi(Z,T,\lambda,m) = -2(\lambda-\lambda_1)(\lambda-\overline{\lambda}_1)Z + \Theta(T,\lambda,m) \tag{99}$$

and F is given by (83).

7. Conclusions

In conclusion, by means of the direct method, we have presented the explicit expressions of the approximation solution of the periodic amplification of a soliton in an optical fibre link with loss. The approximation solution consists of two portions. One portion is the adiabatic solution (75) which is a slowly varying portion in the transmission of the light wave and another portion is the first-order correction term (88) which is a small and rapidly varying portion.

In order to observe the behaviour of the approximation solution directly, we present one typical example for the case of non-resonance [14] according to formulae (3), (75) and (88). Parameters adopted here are: loss rate $\gamma = 0.028\,\text{km}^{-1}$, dispersion distance $z_0 = 404.93$ km, $\Gamma = 11.183$, $Z_A = 0.125$, $\nu_0 = 2$ and $\mu_0 = \tau_{01} = \varphi_{01} = 0$. The results are shown in figures 1 and 2. Figure 1 is the behaviour of the amplitude $|u(Z,T)|$ of the approximation solution for equation (1), which shows that its propagation is periodically stable except for the small tail oscillation resulting from the noise of the amplifier, although we have the distance of transmission 1000 km. The intensity of this tail oscillation can be clearly seen from figure 2

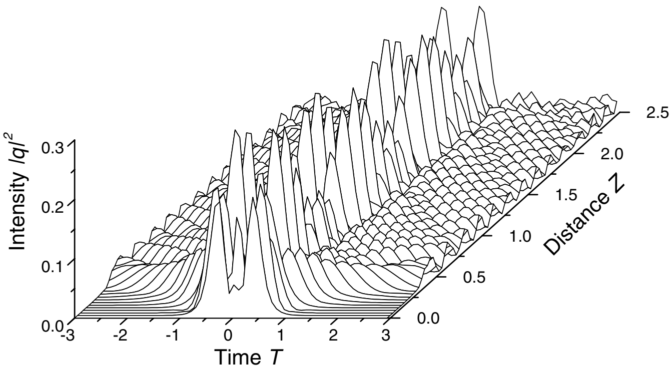

Figure 2. The behaviour of the intensity $|q(Z, T)|^2$ of the correction term, the parameters are the same as in figure 1.

which is the behaviour of the intensity $|q(Z, T)|^2$ of the correction term which shows that the correction term is a small rapidly varying quantity. For the effect of resonance, it has been discussed by using the Lie transformation and averaging method in [14].

Furthermore, the present direct method can be used to investigate the system of dispersion management with periodically varying dispersion [20] and other fields.

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